On the Microfoundations of Aggregate Demand and Aggregate Supply

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Abstract

Economies with nominal rigidities are usually modeled in an Aggregate Demand/Aggregate Supply framework. Early papers in the 'New Keynesian Economics' literature typically assumed aggregate demand followed the quantity theory of money, but provided much more detailed formulations of aggregate supply. More recent papers have tried to give more detailed microfoundations for aggregate demand. However, since consumers and firms are linked both through the goods market and the labor market, in general the form of aggregate demand is unlikely to be independent of the specification of production technology and wage- and price-setting behavior which underlie aggregate supply, and aggregate supply may also not be independent of the preferences and constraints which underlie aggregate demand. This paper establishes necessary and sufficient conditions for a broad class of models under which aggregate demand may be derived separately from aggregate supply. The conditions are that both the marginal rate of substitution of consumption across periods and the marginal rate of substitution between consumption and real balances must be independent of hours worked. These conditions are restrictive; they are violated by two common money demand models, the Baumol-Tobin model and the shopping time model.

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1 Introduction

It is extremely common to model economies with nominal rigidities in an Aggregate Demand/Aggregate Supply framework. Aggregate Demand is a downward-sloping relationship between output and the price level, which is usually either derived from more basic assumptions about demand for goods and money or assumed to follow the quantity theory of money. Aggregate Supply is a relationship among output and prices with varying slope which is usually derived from specifications or production, wage- and price-setting decisions by firms. This approach is followed by most leading Principles and Intermediate Macroeconomics textbooks[1]. It is also characteristic of many early papers in the New Keynesian economics literature, which typically have very detailed specifications for firms' price-setting decisions underlying aggregate supply, but assume the quantity theory or some other simple specification of aggregate demand [2]. Even more recent papers, which have considered the consequences of combining nominal rigidities with dynamic general equilibrium models, usually derive aggregate demand and supply separately. [3]

There were previous attempts to provide microeconomic foundations for Keynesian results: the 'Disequilibrium' literature of Barro and Grossman (1971,1976), Malinvaud(1977) and Benassy (1986). That literature imposed exogenously fixed wages and prices on small general equilibrium models, and focused on the effects of the consequent rationing of consumers or firms. A key insight was that rationing in one market was likely to affect supplies or demands in other markets because consumers and firms are linked in more than one market (i.e. the labor and goods markets).

While the New Keynesian literature has in some respects superseded the Disequilibrium literature by endogenizing price rigidity, it has ignored the general equilibrium spillovers across markets by deriving aggregate demand and supply separately. In general, the production, wage and price-setting specifications of firms are likely to affect consumers' demand for goods through their effects on the labor market. A clear example is that involuntary unemployment caused by nominal wage rigidity, as in the contracting

[1]For example, Mankiw(1997a,b), Samuelson and Nordhaus(1997), Abel and Bernanke(1997), Blanchard(1997).

[2]See for example Mankiw(1985), Fischer(1977), and other articles collected in Mankiw and Romer(1991).

[3]Examples include Fuhrer (1997), McCallum and Nelson(1999), Chari, Kehoe and McGrattan (1996), Goodfriend and King (1997), Kimball (1995) and Ireland (2000).

models of Fischer (1977) and Taylor (1979), is likely to alter the demand for goods through substitution between consumption and leisure. Hence aggregate demand cannot generally be derived separately from aggregate supply. The current practice of doing so is comparable to the approach of assembling general equilibrium models from mutually contradictory partial equilibrium models criticized by Brainard and Tobin(1968) and Sims (1980).[4]

This paper shows that one can specify conditions on preferences under which the aggregate demand curve derived from a standard general equilibrium model is invariant to the assumptions about wage and price setting underlying aggregate supply; for some specifications of aggregate supply, it is also possible to derive an aggregate supply specification independently from aggregate demand. The two necessary and sufficient conditions are that both the marginal rate of substitution between consumption across periods and between consumption and real balances within periods be independent of leisure. These restrictions eliminates spillovers between the markets for goods and labor which would otherwise mean that both the supply and demand for goods would depend on conditions in the labor market. In static models, only the latter condition is needed. These conditions are somewhat restrictive, as they are inconsistent with two common money-demand models: the Baumol-Tobin model when transactions costs include the opportunity cost of time, and shopping-time models.

The paper proceeds as follows: after the introduction, Section 2 sets up a simple general equilibrium model and proves general conditions under which aggregate demand and supply can be derived separately for both static and dynamic models. Section 3 provides a simple demonstration of how altering preferences to make the conditions hold can alter the result of the model. Section 4 discusses how restrictive the conditions are, Section 5 extends the results to some interesting special cases, and Section 6 concludes.

2 Aggregate Demand and Supply: General Conditions

The following two subsections discuss, respectively, conditions under which it is possible to derive aggregate demand without reference to specification

[4] Barro(1995), Colander (1995) and Fields and Hart (1994,1995) have criticized these models on a related point, discussed below.

of the firm's problem and conditions under which it is possible to derive aggregate supply without reference to specification of the consumer's problem. For simplicity, I begin with a dynamic model with a single consumption good under certainty, and defer consideration of the case of uncertainty or multiple consumption goods to section 5. Following much of the literature, all models here will abstract from capital. With capital, the presence of investment demand as part of aggregate demand and of capital in the production function underlying aggregate supply would inseparably link the two equations and make this exercise impossible.[5]

2.1 Deriving Aggregate Demand Independently of Aggregate Supply

There is a representative consumer who solves the following standard maximization problem:

$$\max_{\{C_t\},\{L_t\},\{M_t\},\{B_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t, 1-L_t, \tfrac{M_t}{P_t}) \quad (1)$$

$$s.t. \quad C_t + \tfrac{B_t}{P_t} + \tfrac{M_t}{P_t} \leq \tfrac{W_t}{P_t} L_t + \Pi_t + (1+i_{t-1})\tfrac{B_{t-1}}{P_t} + \tfrac{M_t - 1}{P_t} + T_t \quad (2)$$

$$L_t \leq 1. \quad (3)$$

In words, the consumer chooses a sequence of consumption C_t, hours worked L_t, nominal balances M_t and nominal (government) bond holdings B_t to maximize utility (1) subject to the flow budget constraint (2) and time endowment constraint (3). Π_t denotes profits made by firms, owned by the consumer, i_{t-1} is the nominal interest rate on nominal bond holdings at time $t-1$, and T_t represents transfers made by the government. The latter faces its own flow budget constraint:

$$T_t = \frac{M_t - M_{t-1}}{P_t} \quad (4)$$

i.e. monetary transfers are equal to seignorage[6].

The consumer takes the sequences of nominal interest rates i_t, prices P_t, nominal wages W_t and profits Π_t as given. Production is done by firms,

[5] Unless one were to ignore capital in the production function, but not investment demand.

[6] One could include government purchases of goods and services G_t without altering subsequent results. The current formulation is chosen for simplicity.

which are owned by the consumer. Specification of firms' behavior will allow the determination within the model of output Y_t, prices P_t, nominal wages W_t and profits Π_t. Only the sequence of nominal interest rates i_t and the policy variables B_t and M_t will be exogenous to the full model; for simplicity, a money supply rule is omitted. For now, the nature of production and the competitive environment will be unspecified. The national income accounting identity is $Y_t = C_t$.

Define an aggregate demand relationship as a relationship among Y, P and the policy variable M which is derived from the consumer's optimization problem. An aggregate supply relationship is a relationship between Y and P and possibly W derived from the firm(s)' optimization problem. The following proposition establishes necessary and sufficient conditions on the consumer's preferences under which the aggregate demand relationship is invariant to specification of the firm's optimization problem.

Proposition 1 *For the model defined by (1)-(4) and a specification of firm behavior, necessary and sufficient conditions for an aggregate demand relationship to be invariant to specification of the firm's problem are:*

1. *The marginal rate of substitution between C_t and C_{t+1} is independent of L.*

2. *The marginal rate of substitution between C_t and M_t is independent of L.*

Proof

Sufficiency: Let λ_t and ν_t denote the sequences of Lagrange multipliers for the constraints (2) and (3). The first-order conditions for optimization (not including the transversality condition) are then:

$$\beta^t U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \lambda_t \tag{5}$$

$$\beta^t U_L\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \lambda_t \frac{W_t}{P_t} - \nu_t \tag{6}$$

$$\beta^t U_M\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \lambda_t - \lambda_{t+1}\frac{P_t}{P_{t+1}} \tag{7}$$

$$\lambda_t = \lambda_{t+1}\frac{(1+i_t)P_t}{P_{t+1}}. \tag{8}$$

One can combine the first, third and fourth first-order conditions to obtain:

$$U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \frac{1+i_t}{i_t} U_M\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) \tag{9}$$

and combine the first with itself led one period and the fourth to obtain:

$$U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \frac{\beta P_t(1+i_t)}{P_{t+1}} U_C\left(C_{t+1}, 1-L_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right). \tag{10}$$

The first equation is a standard money demand equation, and the second is a standard intertemporal Euler equation for consumption. Log-linearized versions of these in a model with uncertainty serve as the basis for the IS and LM equations in McCallum and Nelson. (1997)[7]

These can both be rewritten as follows:

$$MRS^{C_t}_{M_t} = -\tfrac{1+i_t}{i_t} \tag{11}$$

$$MRS^{C_t}_{C_{t+1}} = -\tfrac{\beta P_t(1+i_t)}{P_{t+1}} \tag{12}$$

One can then solve the two first-order conditions to obtain:

$$MRS^{C_t}_{M_t} = \frac{MRS^{C_t}_{C_{t+1}}}{MRS^{C_t}_{C_{t+1}} - \frac{\beta P_t}{P_{t+1}}} \tag{13}$$

Finally, since there are no government purchases and no investment, $Y_t = C_t \quad \forall t$. Given the two conditions of the proposition, the above equation represent a nonlinear difference equation in Y, M and P, which is independent of the specification for price and wage setting and the parameters of the production function.

Necessity: Suppose either condition is untrue, or both are. Then equation (13) is no longer purely a function of Y, M and P, but also involves L. There are two ways to solve out for L. The first involves specifying the production technology and inverting it to obtain L as a function of Y and possibly other parameters. But this is not independent of the firm's problem, although it is independent of behavioral assumptions about the firm. The second way is to use the consumer's first order condition, equation (6), to solve for L in terms of Y, M, W and P. Solving out for W will still require specification of labor demand. *Q.E.D.*

[7] It also satisfies the critique of IS-LM models made by King (1993), in that it embodies forward-looking behavior.

In models of this type, firms and workers are connected in two different ways: through the goods market and through the labor market. Since labor demand will be different for different production functions and specifications of wage or price-setting behavior, aggregate demand for goods will also differ across specifications unless hours worked do not affect other decisions by consumers at the margin. A key insight of the first attempts to provide microeconomic foundations for Keynesian results, the 'Disequilibrium' literature of Barro and Grossman (1971, 1976), Malinvaud (1977), Benassy (1986, 1993) and others[8], was that nominal price and wage rigidity, by causing rationing in some markets, would affect behavior in other markets. Thus a fixed nominal wage, by potentially causing rationing of labor supply, could also affect goods demand through the substitution of leisure for consumption.

An easy way to arrive at the two conditions is to assume preferences are additively separable in hours, consumption and money (i.e. $U = V(C, \frac{M}{P}) + X(1-L)$). This is commonly done in the competitive equilibrium business cycle literature (see for example Greenwood, Hercowitz and Huffman (1988)). While it is recognized that making this assumptions allows for simplifications by ruling out certain kinds of cross-market effects, it does not seem to be generally realized that this assumption is necessary for splitting the model into a production and price-setting component and a demand component, particularly in the presence of nominal rigidities. Section 4 explores other implications of making assumptions of this kind.

As noted in the proof, it is possible to obtain an aggregate demand specification which is invariant to the price and wage-setting behavior by simply solving out for hours worked by inverting the production function.[9] However, aggregate demand will vary with the form of the production function, which is not typically the way that aggregate demand is specified.

2.2 Deriving Aggregate Supply Independently of Aggregate Demand

The preceding subsection derived conditions on preferences under which the details of AS (that is, the wage or price setting, production and input choice

[8] Danthine and Donaldson (1995) is a recent dynamic example.

[9] This will not be true in cases where firms are not on their labor demand curves, which can happen in models of price and wage stickiness where the real wage is below the market-clearing level.

decision of firms) were irrelevant to AD. This subsection discusses conditions on the firm's optimization problem under which the details of the derivation of AD are irrelevant to AS.

As the research programs on New Keynesian economics and on the evaluation of monetary policy rules have primarily been focused on devising new approaches to AS, there is no single framework of microfoundations which contains all of them. Below, I consider several common approaches to aggregate supply

2.2.1 Wage Setting

In the models of Fischer (1977), Taylor (1979), Fuhrer and Moore (1995) and many others, aggregate supply consists of:

1. A wage contracting equation. Nominal wages over several periods are set as a function of past and expected future real wages, past, current, and expected future real wages of workers with other contracts, and current and expected future levels of output. This equation is exogenously specified and is not derived from more explicit microeconomic foundations.

2. Either a price-wage markup equation or a labor demand equation. The latter is needed in models in which output does not enter the wage contracting equation, and therefore a link between the real wage and output is needed. The labor demand equation is usually derived for a price-taking, perfectly competitive firm, in which the details of consumer demand do not enter into the optimization process.

In these models as usually written, then, consumer optimization is irrelevant, and hence no conditions on the production function or wage setting are need to ensure that AS can be derived separately from AD.[10]

2.2.2 Exogenous Price Setting

In the first models with nominal rigidities, nominal prices were set exogenously. A key contribution of the New Keynesian economics literature was

[10]If firms were imperfect competitors, then consumer demand would be relevant. See the discussion of endogenous price setting below. Also, if firms are not on their labor demand curve, because the nominal wage is set at a level such that at the resulting real wage, labor demand exceeds labor supply, then the details of labor supply matter.

the endogenization of price stickiness. Small cost of changing prices could lead imperfectly competitive firms to individually resist changing prices, which then implies aggregate price stickiness. Probably the most commonly used specification for price setting is the Calvo(1983)-Rotemberg(1982) formulation, in which there is a constant hazard rate for individual firms to adjust prices, arising from a quadratic cost of changing prices.

The Calvo-Rotemberg formulation has explicit microeconomic foundations, but they are usually not presented; instead, the specification is often given exogenously. Of course, if it truly were exogenous, the details of demand wouldn't matter.[11] But, as the next subsection discusses, if this formulation were rederived for several different specifications of demand, details of the formulation might change.

2.2.3 Endogenous Price Setting

Models with endogenous price setting have an imperfectly competitive firm or firms which choose prices and output to maximize the present value of profits, subject to a production function, consumer demand and a contraint that the loss in profits from changing prices be smaller than the cost of changing prices. One general specification is:

$$\max_{\{P_{it}\},\{Y_{it}\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{P_{it}}{P_t} Y_{it} - \frac{W_t}{P_t} L_{it} - Z\left(\frac{P_{it}}{P_{it-1}}\right) \right) \quad (14)$$

$$s.t. Y_{it} = F(L_{it}) \quad (15)$$

$$Y_{it} = D\left(\frac{P_{it}}{P_t}, \frac{M_t}{P_t}\right), \quad (16)$$

where F is the production function, D the consumer's demand and Z the cost of changing prices.

Clearly, since consumer demand enters into the firm's optimization problem, the details of demand in principle affect the firm's decision. It is possible, particularly in the case of a single good, to take an "off-the-shelf" demand function (e.g. Dixit-Stiglitz or the quantity theory) and use it. Doing so will obscure the role of preferences parameters or taste shocks in aggregate supply.[12] If such shocks are important shifters of aggregate demand, then their effects on aggregate supply may also be of interest.

[11] Except in the cases where the markup between price and marginal cost shrinks to zero, so that output is no longer supplied on demand. This case will be further discussed in section 5.

[12] See Ireland (2000) for a nice example of when such shocks might matter for aggregate supply.

The above formulation for the profit-maximization problem, while frequently used, does not take into the account the possibility of a changing marginal utility of wealth through time. Formulations of the problem which do would take the following form:

$$\max_{\{P_{it}\},\{Y_{it}\}} \sum_{t=0}^{\infty} \beta^t U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right)\left(\frac{P_{it}}{P_t}Y_{it} - \frac{W_t}{P_t}L_{it}\right) \quad (17)$$

$$s.t. Y_{it} = F(L_{it}) \quad (18)$$

$$Y_{it} = D\left(\frac{P_{it}}{P_t}, \frac{M_t}{P_t}\right), \quad (19)$$

This approach adds an additional way in which the details underlying aggregate demand matter: the marginal utility of consumption term, U_C. As shown in Ireland (2000), this term will help contribute to the introduction of preference parameters for money demand unless consumption, money and leisure are additively separable.

2.3 Static Model

In a static model, the conditions under which one can derive aggregate demand independent of specification of the problem of the firm(s) are weaker, and one can more easily get an aggregate demand relationship resembling the conventional quantity theory. Assume consumers solve the following problem, similar to that of section 2.1.

$$U\left(C, 1-L, \frac{M}{P}\right) \quad (20)$$

$$s.t. \quad C + \frac{M}{P} = \frac{M'}{P} + \frac{W}{P}L + \Pi + T \quad (21)$$

$$L \leq 1, \quad (22)$$

where M' denotes initial holdings of nominal money, and notation is otherwise identical to the dynamic model.

The government again faces the budget constraint

$$T = \frac{M - M'}{P} \quad (23)$$

An proposition analogous to Proposition 1 can be derived in this static case:

Proposition 2 *A necessary and sufficient condition for deriving an aggregate demand relationship between Y and P solely from the consumer's optimization problem is that the marginal rate of substitution between consumption and real balances be independent of hours.*

Proof: **Sufficiency**: Let the budget constraint (15) and time endowment constraint (16) have Lagrange multipliers λ and ν. Maximizing utility (14) subject to (15) and (16) implies the following first-order conditions:

$$U_C\left(C, 1-L, \frac{M}{P}\right) = \lambda \tag{24}$$

$$U_L\left(C, 1-L, \frac{M}{P}\right) = \lambda \frac{W}{P} - \nu \tag{25}$$

$$U_M\left(C, 1-L, \frac{M}{P}\right) = \lambda \tag{26}$$

Equating the first and third first-order conditions implies:

$$MRS^C_{\frac{M}{P}} = -1 \tag{27}$$

If the MRS is independent of L, then one can invert it to solve for $C = h(\frac{M}{P})$. Using the national income accounting identity that $Y = C$, one then has $Y = h(\frac{M}{P})$, an aggregate demand relationship between Y and P, as desired. **Necessity**: If the MRS is not independent of L, solving (21) involves either inverting the production function or invoking the first-order condition for L, which would as in the dynamic case involve specifying labor demand. *Q.E.D.*

The following result makes the proposition more operationally useful.

Result 1 *Weak separability of preferences in $(C, \frac{M}{P})$ and L is sufficient but not necessary for the MRS between C and M to be independent of L.*

Proof: If $U = U(V(C, \frac{M}{P}), X(\bar{L} - L))$, then $U_C = U_1 V_1$, $U_M = U_1 V_2$, so $-\frac{U_C}{U_M} = -\frac{V_1}{V_2} = g(C, \frac{M}{P})$.

Hence writing utility as a separable function of consumption, monetary services and leisure allows one to avoid many of the effects of spillovers across markets described above.

3 Application

This section shows how imposition of the conditions affects the results of the model. For simplicity, I use a static setting.

3.1 Conditions Hold

Consider the following simple static model. Preferences are:

$$U\left(C, L, \frac{M}{P}\right) = C^{\alpha}\left(\frac{M}{P}\right)^{\gamma} + (1-L)^{\gamma} \tag{28}$$

The budget constraint is:

$$C + \frac{M}{P} = \frac{M'}{P} + \pi + \frac{W}{P}L + T, \tag{29}$$

where Π denotes profits and T denotes a monetary transfer by the government. The transfer again satisfies the condition $T = \frac{M-M'}{P}$.

It is easy to see that condition of Proposition 2 holds, and that one can solve for the level of aggregate demand simply by looking at the consumer's problem, without considering the nature of firm behavior. To do so, note that the first-order conditions for maximization are:

$$\alpha C^{\alpha-1}\left(\tfrac{M}{P}\right)^{\gamma} = \lambda \tag{30}$$

$$\beta(1-L)^{\beta-1} = \lambda \tfrac{W}{P} - \nu \tag{31}$$

$$\gamma C^{\alpha}\left(\tfrac{M}{P}\right)^{\gamma-1} = \lambda \tag{32}$$

Using the national income accounting identity that $Y = C$, the first and third first-oder condition can be combined to yield:

$$Y^D = \frac{\alpha}{\gamma}\frac{M}{P} \tag{33}$$

Solving for aggregate supply under different conditions requires specification of the firm's problem. For analytical simplicity, assume that there is a representative firm which is perfectly competitive.[13] Production is given by $Y = AL^{\eta}$.

Real profits $\Pi = Y - \frac{W}{P}L$. One can readily show that assuming perfect competition, profit maximization implies labor demand is given by:

$$L^D = \left(\frac{1}{A\eta}\frac{W}{P}\right)^{-\frac{1}{1-\eta}} \tag{34}$$

[13] Driscoll(1998) discusses the relationship between perfectly competitive models with exogenous nominal rigidities and imperfectly competitive models with nominal rigidities. Essentially, assuming the former yields very similar answers to assuming the latter.

3.1.1 Flexible Price Equilibrium

Labor supply is derived from combining the first-order condition for labor with the other first-order conditions. Equating this to labor demand yields:

$$Y^S = \frac{A}{1 + \left(\frac{\gamma^\gamma \alpha^{1-\gamma} A^{\alpha+\gamma}}{\beta}\right)^{\frac{\eta}{1-\beta}}} \tag{35}$$

3.1.2 Fixed Price Equilibrium

When prices are exogenously fixed, at level $\bar{P}$, aggregate supply is simply given by $P = \bar{P}$, and output is demand-determined. [14]

3.1.3 Fixed nominal wage equilibrium

Suppose the nominal wage is fixed at $W = \bar{W}$, but the price level is free to clear the goods market. Given that price, there are two possibilities:

- The real wage is above the level needed to clear the labor market.
- The real wage is below the level needed to clear the labor market.

The first case, which corresponds to labor supply exceeding labor demand, is the more usual case. If it is assumed that a short-side rationing rule is followed, the amount of labor transacted is determined by labor demand, creating involuntary unemployment. This is the approach followed by the contracting models of Fischer (1977) and Taylor (1979). In this case, aggregate supply is given by:

$$Y^S = \left(\frac{1}{\eta A^{\frac{1}{\eta}}} \frac{\bar{W}}{P}\right)^{-\frac{\eta}{1-\eta}}, \tag{36}$$

which is upward-sloping.

[14] This is of course true only if the price level $\bar{P}$ is at or above the level which clears the market. Below that level, as in the disequilibrium literature, the level of output will be determined by supply considerations. Consumers will then face another constraint, on the demand for goods. With imperfect competition, the price level never falls below the market-clearing level because of the presence of a markup of price over marginal cost. See Driscoll (1998) for details.

3.1.4 Upward-Sloping Aggregate Supply

The previous sections show than one can derive horizontal, vertical and upward-sloping aggregate supply relationships independent of the specification for aggregate demand. Barro (1994), Colander (1995) and Fields and Hart (1990, 1991) have all claimed that an upward-sloping aggregate supply is inconsistent with a traditional aggregate demand relationship, except in the case of nominal wage rigidity.[15] The heart of their arguments, which draw on the Disequilibrium literature, is that aggregate demand, with its usual 'Keynesian multiplier', is derived under the assumptions that prices are fixed and output is demand determined. But an upward-sloping aggregate supply curve implies that firms are able to choose both price and quantity, which is an apparent contradiction, since output in principle need not move to satisfy demand.

As is argued in Driscoll (1998), this point is true if, at a given price level, the implied consumer demand for goods exceeded what firms were willing to supply. Then, producers would truly not be willing to accommodate demand, and one would have to rederive aggregate demand under the constraint that demand be constrained by supply. However, in a monopolistically competitive setting, though, firms are able to choose both price and quantity, and will choose them so that consumers are always on their demand curves. In a sense, output is still demand-determined, but the level of demand is also adjustable. Appendix A provides a simple example of how an upward-sloping aggregate supply relationship may be obtained in an imperfectly competitive setting by modifying a model used in Driscoll (1998)

3.2 Conditions Do Not Hold

Now let preferences be defined as:

$$U\left(C, L, \frac{M}{P}\right) = C^{\alpha}(1-L)^{\beta} + \left(\frac{M}{P}\right)^{\gamma}, \tag{37}$$

The budget constraint, as before, is:

$$C + \frac{M}{P} = \frac{M'}{P} + \Pi + \frac{W}{P}L + T, \tag{38}$$

[15]Which Barro (1995) dismisses as implausible because of the well-known countercyclical real-wage implication.

where government transfers, $T = \frac{M-M'}{P}$. The production function is $Y = AL^{\eta}$, and firms are again perfectly competitive.

Again letting λ and ν be the Lagrange multipliers for the budget constraint and time-endowment constraint,

$$\alpha C^{\alpha-1}(1-L)^{\beta} = \lambda \tag{39}$$

$$\beta C^{\alpha}(1-L)^{\beta-1} = \lambda \frac{W}{P} - \nu \tag{40}$$

$$\gamma \left(\frac{M}{P}\right)^{\gamma-1} = \lambda. \tag{41}$$

The key difference from the previous subsection is that the marginal rate of substitution between consumption and real balances now depends on hours, L. Hence solutions for aggregate demand will at the least depend on the specification of the production technology, and alternative solutions for aggregate demand will exist, depending on the wage and price-setting specification. This is shown below for three cases: flexible prices, fixed prices and fixed nominal wages.

3.2.1 Flexible Prices:

Combining the first-order conditions for consumption and real balances yields:

$$Y = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}} (1-L)^{\frac{\beta}{1-\alpha}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-\alpha}} \tag{42}$$

From here, there are two equivalent ways to solve for the level of aggregate demand. The first, which is valid regardless of the specification of price- and wage-setting, is to invert the production function to solve out for hours. Doing so yields:

$$Y^D = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}} \left(1 - \left(\frac{Y}{A}\right)^{\frac{1}{\eta}}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-\alpha}}, \tag{43}$$

which implicitly gives aggregate demand as a function of M and P.

A second way is to use the other first-order condition to solve out for $1-L$ (i.e. implicitly using labor supply. Doing so yields:

$$Y^D = \left(\frac{\gamma}{\alpha^{1-\beta}\beta^{\beta}}\right)^{-\frac{1}{1-(\alpha+\beta)}} \left(\frac{W}{P}\right)^{-\frac{\beta}{1-(\alpha+\beta)}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-(\alpha+\beta)}} \tag{44}$$

Since firms are still on their labor demand curves,

$$L^D = \left(\frac{1}{A\eta}\frac{W}{P}\right)^{-\frac{1}{1-\eta}} \tag{45}$$

Substituting this back into the production function yields:

$$\frac{W}{P} = \eta A^{\frac{1}{\eta}} Y^{\eta-1}. \tag{46}$$

Substituting this back into the expression for aggregate demand yields the following:

$$Y^D = \left(\frac{\alpha^{1-\beta}\beta^{\beta}}{\eta^{\beta} A^{\frac{\beta}{\eta}}\gamma}\right)^{\frac{1}{1-(\alpha+\frac{\beta}{\eta})}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-(\alpha+\frac{\beta}{\eta})}}, \tag{47}$$

which even yields a quantity-theoretic aggregate demand curve if $\gamma = \alpha + \frac{\beta}{\eta}$.

3.2.2 Fixed Prices

Again suppose that the price level is fixed at $P = \bar{P}$, a level above that which clears the goods market. This of course trivially gives aggregate supply.

The first method of solving for aggregate demand outlined above, that of inverting the production function and using it to solve for hours worked, is still possible, so that:

$$Y^D = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}} \left(1 - \left(\frac{Y}{A}\right)^{\frac{1}{\eta}}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-\alpha}}. \tag{48}$$

However, the second way, of using labor demand, is no longer valid. The reason is that labor demand is no longer given by equation 39. Firms are constrained on the goods market to produce at the level of demand. Hence they will not be willing to hire any more labor then is necessary to produce at the given level of aggregate demand. Labor demand is thus given by inverting the production function:

$$L^D = \left(\frac{Y^D}{A}\right)^{\frac{1}{\eta}}, \tag{49}$$

which is no longer directly a function of the real wage.[16] One can solve out for the real wage using labor supply (since consumers are still on their labor supply curves), and then substitute in the new expression for labor demand, but this yields the same expression for aggregate demand given above.

3.2.3 The Case of Contractionary Technology Shocks

Basu, Fernald and Kimball (1997) have noted that in the traditional sticky price model, positive technology shocks (i.e. increases in A) lead to contractions in inputs and no short-run effects on output. The intuition is simple: if prices are unchanged, technology shocks leave the quantity-theoretic level of aggregate demand unchanged. Since $Y = AL$, if A increases and Y is unchanged, L must decrease; firms produce the same amount with less labor. They go on to show that in a model in which money demand depends on a nominal interest rate and there are adjustment costs to investment, output can also decline in the short run.

If consumption and leisure are nonseparable, however, I have shown in the previous section that the productivity disturbance enters both the aggregate demand and aggregate supply expressions. Since it enters the aggregate demand expression positively, the technology shock in fact does raise the level of output. From (48), the rise in demand is less than one-for-one with the shock, so there is still a net decline in the amount of labor, but less so than the separable theory implies.

3.2.4 Fixed Nominal Wage

Now suppose, as in section 3.1.3, that the nominal wage is fixed at a level $W = \bar{W}$. Assume that at the price level which clears the goods market, the corresponding real wage is above the level which clears the labor market.

In this case, consumers are rationed in the amount of hours they can work (i.e. there is involuntary unemployment or underemployment). If we let $\bar{L}$ denote the level of labor demand when $W = \bar{W}$ and the price level is at its goods-market-clearing level, then the time endowment constraint $L \leq 1$ is replaced with $L \leq \bar{L}$ and is binding, so that the Lagrange multiplier $\nu \neq 0$.

The first method of deriving an aggregate demand relationship remains valid, since it only relies on the first-order conditions for consumption and

[16] It is indirectly to the extent that the real wage affects the demand for goods Y^D.

real balances and the production function. Thus,

$$Y^D = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}} \left(1 - \left(\frac{Y}{A}\right)^{\frac{1}{\eta}}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-\alpha}}. \tag{50}$$

The second method is no longer valid, because labor supply is now restricted. A variant of the first method is now valid, however, since the nominal wage is no longer an endogenous variable. We can substitute in labor demand into the expression to obtain

$$Y^D = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}} \left(1 - \left(\frac{1}{A\eta}\frac{\bar{W}}{P}\right)^{\frac{1}{\eta-1}}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{M}{P}\right)^{\frac{1-\gamma}{1-\alpha}}. \tag{51}$$

Aggregate supply is given by inserting labor demand into the production function to yield:

$$Y^S = (\frac{1}{A\eta}\frac{\bar{W}}{P})^{\frac{\eta}{\eta-1}}. \tag{52}$$

Thus changing a seemingly innocuous assumption about preferences leads to aggregate demand specifications which all depend on specification of the production technology and which can also differ across specifications of wage or price setting behavior. In particular, the standard quantity-theoretic aggregate demand expression derived in the flexible-price model, equation 41, is not valid in the fixed price or wage cases. The following sections discuss some common cases in which these issues are likely to be problematic.

4 Are the Conditions Restrictive?

The previous section showed that not imposing the conditions can lead to quite different results. If the conditions are not restrictive, then one can always impose them with impunity. This section argues that they can be restrictive by presenting several commonly-used models they are inconsistent with.

4.1 Money Demand: The Baumol-Tobin model

Real balances enter the utility function to reflect the monetary services they provide. Feenstra (1986) provides conditions under which writing a model with money in the utility function is equivalent to writing a model with a more precise specification of monetary services.

As a specific example he uses the Baumol-Tobin inventory model of cash-management. In that model, if the fixed cost of converting illiquid assets to cash is k, one can write down the standard model as having utility $U(C)$ solely defined over consumption and having transactions costs $k\frac{C}{2M}$ appear in the budget constraint. Equivalently, one can rewrite the model by redefining consumption to equal $C + k\frac{C}{2M}$ and allowing money to enter the utility function, so that utility is now $V(C + k\frac{C}{2M}, M)$.

In calibrating k, it is standard practice to assume that it reflects the opportunity cost of time. This implies that k depends on $1-L$, the amount of leisure. If this is true, then consumption, leisure and money are not separable in the utility function. Hence the conditions of Proposition 1 will be violated, and aggregate demand and supply will not be separately derivable.

4.2 Money Demand: Shopping Time Models

Another common way of modeling money demand is through a 'shopping time' model, as in Brock(1974), or McCallum and Goodfriend(1987). In such models, consumption requires expenditure of time (as is also the case in Ghez and Becker (1975)). Real balances allow economization of shopping time, so that the time endowment constraint in the model of section 2.1 is now:

$$L_t + g\left(C_t, \frac{M_t}{P_t}\right) \leq 1 \qquad (53)$$

As is shown in, for example, Walsh (1998), this constraint can be directly substituted into the felicity function to yield:

$$U = U(C_t, 1 - L_t - g\left(C_t, \frac{M_t}{P_t}\right). \qquad (54)$$

It is clear that this does not satisfy the conditions of Propositions 1 or 2, since the marginal rate of substitution between consumption and real balances by definition is not independent of hours worked.[17]

[17] This assumption is used to justify the inclusion of real balances in the utility function in McCallum and Nelson (1999).

4.3 Consumption and Labor Supply

If the somewhat stronger assumption of weak separability between hours worked and consumption and real balances is imposed, consumption and labor supply decisions become separate. This contradicts the framework of one commonly cited work on labor supply, Ghez and Becker (1975), in which it is assumed that consumption services are produced by combinations of quantities of goods with time. Since the choice of leisure and consumption are not separate, neither is the choice of labor supply and consumption. However, separability is consistent with the conditions required on preferences to be consistent with steady-state growth derived by King, Plosser and Rebelo (1988).

4.4 Aggregate Demand and Supply Disturbances

It is very common in the empirical literature to distinguish impulses as either being to aggregate demand or aggregate supply. In a typical example, Blanchard and Quah (1989) assume that aggregate supply disturbances are permanent, but aggregate demand transitory, making the two orthogonal. While it is generally recognized that permanent changes in TFP will have both supply and demand effects (by altering permanent income), in general even transitory productivity disturbances will affect both labor demand and supply and the supply of and demand for goods.[18] It could be the effects on the 'demand side' of disturbances traditionally assumed to be on the supply side are small, but this should be systematically explored, and not ruled out *a priori*.

5 Extensions

5.1 Multiple Consumption Goods

Many models with endogenous price-setting have a monopolistically competitive final goods sector. This subsection shows that the results of the previous section for a single good are easily extended to when preferences are derived over multiple goods.

[18] This is still true even if technical change is smooth in the aggregate or always non-negative.

5.2 Aggregate Demand: Conditions on Preferences

Assume consumers solve the following problem:

$$\max_{\{C_{it}\},\{L_t\},\{M_t\},\{B_t\}} \sum_{t=0}^{\infty} \beta^t U(\{C_{it}\}, 1-L_t, \tfrac{M_t}{P_t}) \tag{55}$$

$$s.t. \quad \sum_{i=1}^{N} \tfrac{P_{it}}{P_t} C_{it} + \tfrac{B_t}{P_t} + \tfrac{M_t}{P_t} \leq \tfrac{W_t}{P_t} L_t + \Pi_t + (1+i_{t-1}) \tfrac{B_{t-1}}{P_t} + \tfrac{M_t - 1}{P_t} + T_t \tag{56}$$

$$L_t \leq 1, \tag{57}$$

where there are N goods indexed by i, consumption of each good at time t is given by C_{it}, the nominal price of each good is given by P_{it}, and there is some price indexed which is a function of the prices $P_t = g(\{P_{it}\})$. The government's budget constraint, describing the helicopter drop of transfers is again:

$$T_t = \frac{M_t - M_{t-1}}{P_t} \tag{58}$$

We obtain the following set of $N+3$ first-order conditions:

$$\beta^t U_{C_j}(\{C_{it}\}, 1-L_t, \frac{M_t}{P_t}) = \frac{P_{jt}}{P_t} \lambda_t, \quad \forall j \tag{59}$$

$$\beta^t U_L(\{C_{it}\}, 1-L_t, \frac{M_t}{P_t}) = \lambda_t \frac{W_t}{P_t} - \nu_t \tag{60}$$

$$\beta^t U_M(\{C_{it}\}, 1-L_t, \frac{M_t}{P_t}) = \lambda_t - \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{61}$$

$$\lambda_t = \lambda_{t+1} \frac{(1+i_t)P_t}{P_{t+1}}, \tag{62}$$

which are identical to the conditions derived in section 2.1, with the exception of the first set of N equations.

Using the same approach as in section 2.1, we can derived the following two sets of intra- and intertemporal optimality conditions:

$$U_{C_{jt}} = \beta \frac{P_{jt}}{P_{jt+1}} (1+i_t) U_{C_{jt+1}} \tag{63}$$

$$U_M = \left(\frac{i_t}{1+i_t} \right) \frac{P_{jt}}{P_t} U_{C_{jt}}, \tag{64}$$

which we can again combine to yield:

Using the condition that $Y_{jt} = C_{jt}$, this expression is a nonlinear difference equation in Y_{jt},M,P_{jt} and P_t. This can be combined with an appropriate aggregation equation to derive an expression linking Y, M and P.

5.3 Uncertainty

The setup of the problem remains unchanged under uncertainty; the only difference is the addition of an expectations operator in front of the maximand:

$$\max_{\{C_t\},\{L_t\},\{M_t\},\{B_t\}} E_0 \textstyle\sum_{t=0}^{\infty} \beta^t U(C_t, 1-L_t, \frac{M_t}{P_t}) \tag{65}$$

$$s.t. \quad C_t + \tfrac{B_t}{P_t} + \tfrac{M_t}{P_t} \leq \tfrac{W_t}{P_t} L_t + \Pi_t + (1+i_{t-1})\tfrac{B_{t-1}}{P_t} + \tfrac{M_t - 1}{P_t} + T_t \tag{66}$$

$$L_t \leq 1. \tag{67}$$

It is convenient in this case to simply work with two intra- and intertemporal Euler equations directly:

$$U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \beta E_t \left\{ \frac{P_t(1+i_t)}{P_{t+1}} U_C\left(C_{t+1}, 1-L_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)\right\} \tag{68}$$

$$U_C\left(C_t, 1-L_t, \frac{M_t}{P_t}\right) = \frac{1+i_t}{i_t} U_M\left(C_t, 1-L_t, \frac{M_t}{P_t}\right). \tag{69}$$

One can combine these to show that:

$$\left(\frac{MRS_C^M}{MRS_C^M - 1}\right) E_t \left(\frac{P_t}{P_{t+1}} MRS_{C_t}^{C_{t+1}}\right) = 1, \tag{70}$$

so that sufficient and necessary conditions for deriving an aggregate demand relationship are that the marginal rate of substitution between consumption and real balances be independent of labor, and the expectation of the product of the intertemporal marginal rate of substitution for consumption and the rate of inflation be independent of labor. As under certainty, weak separability between labor and the pair of consumption and money is sufficient to ensure this condition.

5.4 Cash-In-Advance

A common alternative to putting money into the utility function is to assume a cash-in-advance constraint. If that constraint is always binding, then in a model with certainty and one good, aggregate demand is always given by $Y_t = \frac{M_t}{P_t}$, i.e. a strict version of the quantity theory with constant unit velocity[19].

[19]Under uncertainty, Svensson (1985) has shown that there is a precautionary demand for money which may lead to a non-unit velocity. When the cash-in-advance constraint

In models in which not all goods are subject to the cash-in-advance constraint, such as Lucas and Stokey (1989), it is still possible to derive a proposition similar to Proposition 1 under which aggregate demand and supply are separable. Below, we consider such a model. Assume consumers solve the following problem. C_{1t} is the cash-only good, and C_{2t} is the credit good.

$$\max_{\{C_{1t}\},\{C_{2t}\}\{L_t\},\{M_t\},\{B_t\}} \textstyle\sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}, 1 - L_t) \tag{71}$$

$$s.t. \quad \tfrac{P_{1t}}{P_t} C_{2t} + \tfrac{P_{1t}}{P_t} C_{2t} + \tfrac{B_t}{P_t} + \tfrac{M_t}{P_t} \leq \tfrac{W_t}{P_t} L_t + \Pi_t + (1 + i_{t-1}) \tfrac{B_{t-1}}{P_t} + \tfrac{M_t - 1}{P_t} + T_t \tag{72}$$

$$L_t \leq 1 \tag{73}$$

$$\tfrac{P_{1t}}{P_t} C_{1t} \leq \tfrac{M_{t-1}}{P_t} + T_t \tag{74}$$

The government's budget constraint is again:

$$T_t = \frac{M_t - M_{t-1}}{P_t} \tag{75}$$

Let μ_t represent the Lagrange multiplier on the cash-in-advance constraint, and all other notation be standard. The first-order conditions for optimization are:

$$\beta^t U_{C_1}(C_{1t}, C_{2t}, 1 - L_t) = \frac{P_{1t}}{P_t}(\lambda_t + \mu_t) \tag{76}$$

$$\beta^t U_{C_2}(C_{1t}, C_{2t}, 1 - L_t) = \frac{P_{2t}}{P_t}\lambda_t \tag{77}$$

$$\beta^t U_L(C_{1t}, C_{2t}, 1 - L_t) = \lambda_t \frac{W_t}{P_t} - \nu_t \tag{78}$$

$$\lambda_t = (\lambda_{t+1} + \mu_{t+1}) \frac{P_t}{P_{t+1}} \tag{79}$$

$$\lambda_t = \lambda_{t+1} \frac{(1 + i_t) P_t}{P_{t+1}}. \tag{80}$$

Suppose the cash-in-advance constraint binds, so that $C_{1t} = \frac{M_t}{P_{1t}}$. By combining the first-order conditions for C_{1t},C_{2t}, M_t and B_t, one can obtain the result that:

$$\beta \frac{P_t}{P_{t+1}} \frac{P_{1t}}{P_{2t}} MRS_{C_{1t}}^{C_{1t+1}} = MRS_{C_{1t}}^{C_{2t}} \tag{81}$$

fails to bind, the standard intertemporal Euler equation for consumption still holds. Independence of the precautionary money demand and of the Euler equation from hours worked should be sufficient conditions for a separable aggregate demand.

If the two marginal rates of substitution are independent of hours worked, the above represents a difference equation in $\frac{M}{P}$,C_2 and P. Hence aggregate demand $Y = \frac{P_1}{P}C_1 + \frac{P_2}{P}C_2 = \frac{M}{P_1} + C_2$ is only a function of M and P.

5.5 Zero Markup

In the discussion of models with exogenously set prices (section 2.2.2), I briefly noted that a necessary assumption for separability of aggregate demand and supply was that the exogenously fixed price level exceeds marginal cost. If this fails to hold, so that the markup is reduced to zero, then the implication that output is supplied on demand also fails.

This failure adds an additional set of constraints to the consumer's problem presented in (1) through (3):

$$C_t \leq \bar{Y}, \tag{82}$$

where $\bar{Y}$ represents the level of output at which price is equal to marginal cost.

If we denote the Lagrange multipliers for this constraint as μ_t, we see that adding it changes the first order condition (5) to the following:

$$\beta^t U_C\left(C_t, 1 - L_t, \frac{M_t}{P_t}\right) = \lambda_t + \mu_t. \tag{83}$$

If price is always above marginal cost, $\mu_t = 0 \forall t$, and we have the same intratemporal optimality condition as before. If this condition fails to hold in any period, the derivation of aggregate demand will be affected in that period and the immediately preceding period. This arises from the failure of the consumption Euler equation to hold exactly, and is very similar to the case of liquidity constraints.

6 Conclusion

This paper has shown conditions under which it is possible in a small general equilibrium model to derive aggregate demand and supply relationships independently from one another. One can derive aggregate demand separately from aggregate supply if:

1. In a dynamic model, assumes the marginal rate of substitution between consumption across periods, and between consumption and real balances within periods, is independent of hours worked.

2. In a static model, assumes that the marginal rate of substitution between consumption and real balances is independent of hours worked.

3. In a cash-in-advance model, assume the constraint binds.

Furthermore, one can always derive an aggregate demand relationship which is independent of the wage and price-setting decisions which underlie aggregate supply. Doing so requires that aggregate demand depend on parameters in the production function, such as productivity, which is not usual. Aggregately supply can only be derived completely independently of aggregate demand if either prices or wages is set exogenously. If they are set endogenously, using an "off-the-shelf" aggregate demand expression in imperfectly competitive firms' decision problems has the cost of obscuring the dependence of aggregate supply on preference parameters.

These conditions are strong. They are inconsistent with two standard money demand models: a version of the Baumol-Tobin model and the shopping-time model. They are also inconsistent with standard models of labor supply in which consumption and leisure are complements. More generally, there are also other ways in which the interaction across markets could lead to differences in aggregate demand and structural relationships aside from the ones highlighted here. A true model of unemployment, in which some consumers are rationed to zero labor supply while others are unconstrained, rather than the model of underemployment presented above would imply that the goods demand of the rationed consumers would be affected through the budget constraint. This is a form of non-representative agent model, and it is quite possible that the aggregated demand curve will depend on the constraint. Models in which consumers and firms interact in yet other markets, such as the capital market, are also likely to have such spillover effects. In a dynamic model with intertemporally-dependent preferences, consumers may respond to potential future rationing in the labor market or firms to rationing in the goods market.

The observation that there are likely to be spillovers between the 'demand side' and the 'supply side' of the model is not new. It was a key element of the 'disequilibrium' literature of Barro and Grossman (1971, 1976) et. al. Such models were (rightly) criticized for the exogeneity of the price and wage stickiness and for being static, and were largely superseded by the 'New Keynesian' models.[20] Although the latter models can replicate many of the results of the former while making some additions, the key result of the general equilibrium interaction across markets was forgotten. Gordon (1990) has criticized older New Keynesian models for their simplified representations of aggregate demand, compared with the earlier literature. Barro (1995), Colander (1994) and Fields and Hart (1994,1995) have all used the results of that literature to argue that an upward-sloping aggregate-supply formulation is inconsistent with a conventional aggregate demand specification. Deaton and Muellbauer (1980) in a standard graduate microeconomics text have looked at the effects of separable preferences on rationing in a microeconomic context.

As briefly discussed above, even if these conditions are met, aggregate demand and supply will not be independent if output is not demand determined. This may happen in models in which prices are exogenously fixed for some period of time; the effects on aggregate demand are comparable to those caused by the consumption Euler equation's failure to hold in models with credit constraints.

This paper suggests that one can continue to use the standard and convenient Aggregate Demand/ Aggregate Supply formulation. Imposing a quantity-theoretic aggregate demand curve often makes it simple to solve models and makes their inner workings more clear than a more fully specified general equilibrium (or disequilibrium) model. However, since strong assumptions are needed to make this approach valid, it will not be the best approach in all cases. In particular, models which are going to be calibrated or models whose first-order conditions only are going to be estimated do not require as much analytical simplicity. Models which attempt to explain unemployment or involve other labor market behavior may not want to rely on such a strong assumption, either. Even if they do, they should explicitly reflect a conscious choice that the assumptions they make affect their predictions as well as provide analytical simplicity, to avoid what Barro (1995) has termed 'a clear case of technical regress.'

[20] Mankiw (2000) briefly discusses the relationship between the two models.

Appendix A:Upward-Sloping Aggregate Supply

The following example, a variant of Blanchard and Kiyotaki(1987)[21], shows that it is possible to derive an identical aggregate demand relationship under a variety of different assumptions about price and wage determination.

There are N goods and N firms. Consumers' preferences over goods, real balances and leisure are given by:

$$U = (N^{\frac{1}{1-\theta}} C)^{\alpha} \left(\frac{M}{P}\right)^{\gamma} + (\bar{L} - L)^{\beta} \tag{84}$$

where $C = (\sum_{j=1}^{N} C_j^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}}$, $P = (\frac{1}{N}\sum_{j=1}^{N} P_j^{1-\theta})^{\frac{1}{1-\theta}}$ and $L = \sum_{j=1}^{N} L_j$.

The consumer's real budget constraint is:

$$\sum_{j=1}^{N} \frac{P_j C_j}{P} + \frac{M}{P} = \frac{M'}{P} + \sum_{j=1}^{N} \Pi_j + \frac{W}{P} L \tag{85}$$

where $\Pi_j = \frac{P_j}{P} Y_j - \frac{W}{P} L_j$. The production function for each good is $Y = AL_j$.[22]

The first-order conditions for the consumer's optimization are:

$$C_j : \alpha N^{\frac{\alpha}{1-\theta}} C^{\alpha-1} \left(\frac{C_j}{C}\right)^{\frac{-1}{\theta}} \left(\frac{M}{P}\right)^{\gamma} = \lambda \frac{P_j}{P} \tag{86}$$

$$L_j : \beta(\bar{L} - L)^{\beta-1} = \lambda \frac{W}{P} - \nu \tag{87}$$

$$M : \gamma(N^{\frac{1}{1-\theta}} C)^{\alpha} \left(\frac{M}{P}\right)^{\gamma-1} = \lambda \tag{88}$$

Note from the first and third conditions that the marginal rate of substitution between C_j and M is independent of leisure, so that the conditions of Proposition 2 are satisfied. Hence it is possible to derive an aggregate demand relationship independent of the firm's problem. By summing the first-order conditions for consumption and equating it to the first-order conditions for real balances, using the fact that $Y_j = C_j$ for all j and the fact that $Y = \sum_{j=1}^{N} \frac{P_j Y_j}{P}$, we can derive the following familiar aggregate demand relationship:

$$Y^D = \frac{\alpha}{\gamma} \frac{M}{P} \tag{89}$$

[21] The only differences are that I assume constant returns to scale and a competitive labor market. The latter simply implies that if price is ever pushed to equal marginal cost, we will in fact return to the competitive level of output.

[22] Linearity is assumed simply to make aggregation easier.

Aggregate supply may be easily derived under different price-setting and wage-setting conditions. To do so, we first need two additional optimality conditions. Individual demand curves are:

$$Y_i^D = \frac{1}{N}\left(\frac{P_i}{P}\right)^{-\theta}\left(\frac{\alpha}{\gamma}\frac{M}{P}\right) \tag{90}$$

The optimal choice of relative price from profit maximization, given the demand expression above, is:

$$\frac{P_i}{P} = \frac{W}{P}\frac{\theta}{\theta-1}\frac{1}{A}. \tag{91}$$

Aggregate supply is then given by:

- **Flexible Prices and Wages:** $Y^S = \left(\frac{\alpha^{1-\gamma}\gamma^{\gamma}}{\beta}A\frac{\theta-1}{\theta}\right)^{\frac{1}{\beta-1}}$ (from symmetric equilibrium).

- **Endogenously Fixed Prices**

  Suppose that all firms face a cost of changing prices z. For small changes in M, firms will simply accommodate the level of demand at the current price level and not change prices.[23]

  Aggregate supply is then locally horizontal and can simply be written as $P = \bar{P}$.

- **Fixed Nominal Wage**

  Suppose the nominal wage is fixed at some level $W = \bar{W}$, which again implies a real wage above the market-clearing level. The profit-maximization condition still holds, so that in symmetric equilibrium where $P_i = P$, this implies that the price level is given by: $P = \bar{W}\frac{\theta}{\theta-1}\frac{1}{A}$ (the horizontal aggregate supply under fixed prices is an artifact of the assumption of a linear production function; under the assumption of diminishing returns to scale, equation (15) would be a standard downward-sloping labor demand curve).

[23] An exception arises if price is already very close to marginal cost. Firms will then only accommodate demand to the extent that marginal cost is pushed to price. I will rule out this case, in which consumers are effectively rationed on the goods market, by assumption, although it is a potentially important case in the Disequilibrium literature.

- **Upward-Sloping Aggregate Supply** Let S firms have locally inflexible prices, because they face a menu cost z and $N - S$ firms have flexible prices because they face no menu cost. While it is not possible to obtain an analytic solution for Y after a change in the money stock, we can show in which direction Y changes.

  Let M_0 denote the initial money stock, and $M_1 > M_0$ denote the new money stock, P_0 and P_1 denote the initial and new price levels, Y_0 and Y_1 denote the initial and new levels of aggregate output, P_{NS} denote the new relative prices for the firms without sticky prices, and P_{FLEX} denote the new price level if all firms' prices were flexible. Note that both P_{FLEX} and P_0 are proportional to the money stock, and under the assumption above, $P_0 < P_{FLEX}$.

  By definition,

$$P_1^{1-\theta} = \frac{S}{N}P_0^{1-\theta} + \frac{N-S}{N}P_{NS}^{1-\theta} \tag{92}$$

  Now note that since people are always on their demand curves, $Y_0 = \frac{\alpha}{\gamma}\frac{M_0}{P_0}$ and $Y_1 = [S(\frac{P_0}{P_1})^{1-\theta} + (N-S)(\frac{P_{NS}}{P_1})^{1-\theta}]\frac{1}{N}\frac{\alpha}{\gamma}\frac{M_1}{P_1} = \frac{\alpha}{\gamma}\frac{M_1}{P_1}$, from the definition of P_1. Therefore

$$\frac{Y_1}{Y_0} = \frac{\frac{M_1}{P_1}}{\frac{M_0}{P_0}} \tag{93}$$

  Furthermore, since when prices are flexible, P is proportional to M, we can rewrite (43) as:

$$\frac{Y_1}{Y_0} = \frac{P_{FLEX}}{P_1} \tag{94}$$

  Or in other words, output is larger if $P_1 < P_{FLEX}$.

  I will show that $P_1 < P_{FLEX}$ by assuming otherwise and showing that entails a contradiction.

  First, suppose that $P_1 = P_{FLEX}$. Then $Y_1 = Y_0$. Since people are on their labor supply curves,$(\frac{W}{P})_0 = (\frac{W}{P})_1$. Hence, from the price-setting condition, equation (22), $P_{NS} = P_1$. But since $P_0 < P_{FLEX} = P_1$, this contradicts equation (42).

  Next, suppose that $P_1 > P_{FLEX}$. Then $Y_1 < Y_0$. The resulting decline in labor demand implies, through the labor supply relationship, that $(\frac{W}{P})_1 < (\frac{W}{P})_0$. Again, by the pricing condition, $P_{NS} < P_1$. But we know that $P_0 < P_{FLEX} < P_1$, so again the definition of P_1 is not satisfied (since we have P_1 being equal to the weighted average of two components, each less than P_1.

Hence the only possible solution is $P_1 < P_{FLEX}$ (it can be shown that this is consistent with (42) by a similar argument), so $Y_1 > Y_0$.

To show that Y_1 is increasing in S, note that this implies that P_1 is decreasing in S. Suppose instead that P_1 is increasing in S. Then, since P_0 is unchanged when S changes, for P_1 to increase, P_{NS} must increase, and therefore $\frac{P_{NS}}{P_1}$ must increase. From the price setting equation, this must imply that $(\frac{W}{P})_1 > (\frac{W}{P})_0$. But this implies that labor demand must have risen, which will only happen if $Y_1 > Y_0$. But this is contradictory. Therefore, Y_1 is increasing in S.

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